

The Maximum L_q -Likelihood Estimator in Extreme Value Theory

Lo stimatore di Massima L_q -Verosimiglianza nella Teoria dei Valori Estremi

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Riassunto: In questo lavoro, si propone di utilizzare lo stimatore ML_qE di Massima L_q -Verosimiglianza, introdotto da Ferrari e Yang (2007), per la stima dei parametri della distribuzione del Valore Estremo generalizzata e della distribuzione di Pareto generalizzata. L'analisi empirica, condotta mediante simulazioni Monte Carlo, mostra che lo stimatore ML_qE è più efficiente dello stimatore di Massima Verosimiglianza nel caso in cui si voglia stimare la probabilità di un evento estremo, avendo a disposizione un campione di dimensioni limitate.

Keywords: L_q -Likelihood, Extreme Value Theory, Generalized Extreme Value Distribution, Generalized Pareto Distribution, Tail-related risk measures.

1. Introduction

Recently, extreme value theory has found extensive application in many fields, including finance (e.g., see McNeil *et al.* (2005)). The two prevailing approaches for modeling extreme events are the Block Maxima (BM) and Peaks-Over-Threshold (POT) methods, which are based on two parametric limiting distributions. The BM method relies on the Generalized Extreme Value (GEV) distribution to model the maximum value that a variable takes in a given period of time (block), while the POT method exploits the Generalized Pareto distribution (GP) for modeling the exceedances over a certain threshold (McNeil *et al.* (2005)). Although Maximum Likelihood Estimation is the standard approach, often the number of observations available to estimate GEV and GP parameters is too small to guarantee the desirable large sample properties of MLE. To address this issue, we propose to use the Maximum L_q -Likelihood estimator (ML_qE) (Ferrari and Yang (2007)). Empirical results show that the new estimator is more efficient than the standard MLE when BM and POT approaches are employed to estimate the probability of an extreme event.

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2. The Maximum L_q -Likelihood Method

Let $f(x; \beta_0)$ be the probability density function of the GP or GEV distributions, where $\beta_0 \in \Theta$ is the vector of parameters to estimate. Given a random sample $X_1, \dots, X_n$ from $f(x; \beta_0)$, the classical MLE is derived by optimizing the natural logarithm of the likelihood function. Ferrari and Yang (2007) proposed the Maximum L_q -Likelihood estimator (ML_qE), based on Havra and Charvat's entropy function (Havrda and Charvat (1967)). The ML_qE of β is defined as

$$\tilde{\beta}_n(q) = \arg \max_{\beta \in \Theta} \sum_{i=1}^n L_q[f(X_i; \beta)], \quad (1)$$

where

$$L_q(u) = \begin{cases} \frac{u^{1-q} - 1}{1 - q} & \text{if } q \neq 1, \\ \log u & \text{if } q = 1, \end{cases} \quad (2)$$

and q is a positive constant. The parameter q extends the properties of the traditional MLE, introducing a peculiar type of distortion. When q is smaller than 1, the ML_qE successfully trades bias for variance, gaining accuracy for small sample sizes. Conversely, for values of q arbitrarily close to 1, the new estimator recovers the desirable asymptotic properties of MLE.

3. Simulation Study and Empirical Results

The empirical investigation, carried out using Monte Carlo simulations, aims: (i) to compare the performance of the two estimators in terms of Mean Squared Error for several sample sizes; and (ii) to study the role of the distortion parameter q in the performance of ML_qE as the sample size changes. Given a sample size n (ranging from 5 to 200) and tail probability equal to 0.1%, 1% and 5%, we generate 10000 random samples from the GEV and GP distributions and compute the plug-in estimates of the tail probability. We assess the relative efficiency between the two estimators using the ratio of the Monte Carlo Mean Squared Error of MLE and ML_qE . The empirical results show that when the sample size is relatively small and $q < 1$, the ratio is greater than 1, meaning that the ML_qE outperforms the traditional MLE. In contrast, when the sample size is larger, the bias component plays an increasingly relevant role and eventually the ratio becomes smaller than 1. Our results support the asymptotic findings derived by Ferrari and Yang (2007). Finally, we show that ML_qE has smaller prediction error than MLE when estimating tail-related risk measures for different stock market indexes.

References

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